GEOMETRY OF MEASURES AND PREISS RECTIFIABILITY THEOREM.

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In the 30's when Besicovitch studied sets with locally finite lenght, he arose the following question, whether the infinitesimal properties of the length of a set E in the plane yield geometric information on E it self? This question marks the begining of the study of geometry of measures and the field known as geometric measure theory.
In this series of lectures we presents an introduction to the geometry of measures, relating the tangent measures with the regularity of their support, all this aimed to proof the rectifiability theorem of Preiss:

Theorem 1. *Let μ be a locally finite measure on $\mathbb{R}^n$ and α a nonnegative real number. Assume that the following limit exists, is finite and nonzero for μ-a.e. x:*

$$\lim_{r \downarrow 0} \frac{\mu(B(x,r))}{r^\alpha}.$$

Then, either $\mu = 0$, or α is a natural number $k \leq n$. In the latter case, a measure μ satisfies the requirement above if and only if there exists a Borel measurable function f and a countable collection $\{\Gamma_i\}_{i\in\mathbb{N}}$ of Lipschitz k-dimensional submanifolds of $\mathbb{R}^2$ such that

$$\mu(A) = \sum_{i=1}^{\infty} \int_{\Gamma_i \cap A} f dVol^k \quad \forall \text{ Borel set } A,$$

where Vol^k denotes the natural k-dimensional volume measure that a Lipschitz submanifold inherits as a subset of $\mathbb{R}^n$.

1. Lecture 1

The first sesion is aimed to introduce the gownd terminoloy and known facts, at this time we do not prove any statment.

(1) Hausdorff measures.
(2) Rectifiable sets.
(3) Densities of measures.
(4) Weak* convergence.
(5) Covering Theorems & differentiation of measures.
(6) Blow up of measures.

2. Lecture 2

This sesion is aimed to formally introduce the problem.

(1) The structure Theorem.
(2) The four corner Cantor set.
(3) The Besicovitch questions.
(4) Rectifiable measures.

(5) Rectifiable measures: Federer vs. Mattila & Preiss.
(6) Statment of the Theorem.
(7) Historic overview of the problem.

3. Lecture 3

This lecture is devoted to prove the Mastrand's Theorem, where the basic ideas born.

(1) Tangnt measures
(2) Densities & tangent measures
(3) Mastrand's Theorem

4. Lecture 4

In this lecture we give a rectifiable criterion due to Marstrand and Mattila .

(1) Purely unrectifiable sets and projections.
(2) Proof of the Marstrand-Mattila Criterion.

5. Lecture 5

This session is aimed to stablish the Preiss strategy for the main theorem.

6. Lecture 6

Finally we conclude with the overview of the Preiss' proof of the rectifiable theorem.

References

[DL08] Camillo De Lellis. *Rectifiable sets, densities and tangent measures*. 2008.

[Mat99] Pertti Mattila. *Geometry of sets and measures in Euclidean spaces: fractals and rectifiability*, volume 44. Cambridge university press, 1999.

[Pre87] David Preiss. Geometry of measures in r n: distribution, rectifiability, and densities. *Annals of Mathematics*, 125(3):537–643, 1987.

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