Mathematical Research in High School: The PRIMES Experience

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Seriously? Is it really possible for tenth- and eleventh-graders to do original mathematical research?

Yes! Christina and Joseph, as well as over a hundred other students, have done their research at PRIMES (Program for Research In Mathematics, Engineering, and Science: web.mit.edu/primes), which we’ve been running in the MIT mathematics department since January 2011. Every year we receive numerous questions about our program from prospective students and their parents and also from academics who want to organize a similar program. Here we’d like to answer some of these questions, to share our experience, and to tell a wider mathematical community how such a seemingly impossible thing as mathematical research in high school can actually be done.

How do you select projects? Can my student be told to prove the Twin Primes Conjecture in PRIMES?

P.E.: Famous open problems don’t usually make good projects, but we don’t assign “toy projects” with known solutions either. Students delve into real research, with all its uncertainties, disappointments, and surprises. Finding cutting-edge projects requiring a minimal background is one of the trickiest tasks in running PRIMES. Here are some features we want to see in a PRIMES project:

1. **Accessible beginning.** Presence of simple initial steps to get started.

2. **Flexibility.** A possibility to think about several related questions, switching from one to another if stuck, and to tweak the questions if they are too hard or insufficiently interesting.

3. **Computer (experimental) component.** A possibility of computer-assisted exploration aimed at finding patterns and making conjectures. This way students, who often have strong programming skills, can contribute to the project early, when they don’t yet have a working knowledge of the theoretical tools. It is also easier to learn new mathematical concepts, e.g., those from algebra and representation theory, through a hands-on experience with a computer algebra system.

4. **Adviser involvement.** Availability of a research mathematician other than the mentor (usually the professor or researcher who suggested the project) to advise the project through email and occasional meetings. Such meetings make a big difference.

5. **Big picture/motivation.** Connection, at least at the level of ideas, to a wider context and to other people’s work.

6. **Learning component.** The project should encourage the student to study advanced mathematics on a regular basis.

7. **Doability.** A reasonable expectation that a good student would obtain some new results in several months to present at the annual PRIMES conference in mid-May and produce publishable results in one year.

8. Relation to the mentor’s research program or area.
T.K.: A crucial part of research is the art of asking your own questions, not just solving other people’s problems. When the students realize that it is in their power to move the project in a new direction, they get very excited and start feeling ownership of the project. The ability to trust themselves and ask their own questions is very important in their future lives, independent of their career choices. That’s why we try to choose projects that develop this ability.

P.E.: Sounds easy? Well, if you have a bit of free time or have nothing better to do (e.g., during an excruciatingly boring math lecture that you can’t sneak out of), just try to come up with a project satisfying most of these conditions. And when you do, please send it to us!

(...)

Outbreak alert: six students at the Chicago State Polytechnic University in Illinois have been hospitalized with severe vomiting, diarrhoea and stomach pain, as well as wheezing and difficulty in breathing. Some are in critical condition. And the university’s health centre is fielding dozens of calls from students with similar symptoms.

This was the scenario that 17 third-and fourth-year undergraduates dealt with as part of an innovative virology course led by biologist Tammy Tobin at Susquehanna University in Selinsgrove, Pennsylvania. The students took on the role of federal public-health officials, and were tasked with identifying the pathogen, tracking how it spreads and figuring out how to contain and treat it – all by the end of the semester.

Although the Chicago school and the cases were fictitious, says Tobin, “we tried to make it as real as possible.” If students decide to run a blood test or genetic assay, Tobin would give them results consistent with enterovirus D68, a real respiratory virus. (To keep the students from just getting the answer from the Internet, she portrayed the virus as an emergent strain with previously unreported symptoms.) If they decided to send a team to Chicago, Tobin would make them look at real flight schedules and confirm that there were enough seats.

In the end, the students pinpointed the virus, but they also made mistakes: six people died, for example, in part because the students did not pay enough attention to treatment. However, says Tobin, “that doesn’t affect their grade so long as they present what they did, how it worked or didn’t work, and how they’d do it differently.” What matters is that the students got totally wrapped up in the problem, remembered what they learned and got a handle on a range of disciplines. “We looked at the intersection of politics, sociology, biology, even some economics,” she says.

Tobin’s approach is just one of a diverse range of methods that have been sweeping through the world’s undergraduate science classes. Some are complex, immersive exercises similar to Tobin’s. But there are also team-based exercises on smaller problems, as well as simple, carefully tailored questions that students in a crowded lecture hall might respond to through hand-held ‘clicker’ devices. What the methods share is an outcome confirmed in hundreds of empirical studies: students gain a much deeper understanding of science when they actively grapple with questions than when they passively listen to answers.

“We find up to 20% better grades over usual methods,” says Tom Duff, a computer scientist who developed a team-based learning approach at the University of the West of Scotland in Paisley, UK. Other active-learning proponents have found similar gains. Last year, a group led by biologist Scott Freeman at the University of Washington in Seattle published an analysis of 225 studies of active learning in science, technology, engineering and mathematics (STEM) and found that active learning cut course failure rates by around one-third.

“At this point it is unethical to teach any other way,” declares Clarissa Dirks, a microbiologist at the Evergreen State College in Olympia, Washington, and co-chair of the US National Academies Scientific Teaching Alliance, an initiative to reform undergraduate STEM education.

Active learning is winning support from university administrators, who are facing demands for accountability: students and parents want to know why they should pay soaring tuition rates when so many lectures are now freely available online. It has also earned the attention of foundations, funding agencies and scientific societies, which see it as a way to patch the leaky pipeline for science students. In the United States, which keeps the most detailed statistics on this phenomenon, about 60% of students who enroll in a STEM field switch to a non-STEM field or drop out (see ‘A persistence problem’). That figure is roughly 80% for those from minority groups and for women.

Mathematics is a critical part of much scientific research. Physics in particular weaves math extensively into its instruction beginning in high school. Despite much research on the learning of both physics and math, the problem of how to effectively include math in physics in a way that reaches most students remains unsolved. In this paper, we suggest that a fundamental issue has received insufficient exploration: the fact that in science, we don’t just use math, we make meaning with it in a different way than mathematicians do. In this reflective essay, we explore math as a language and consider the language of math in physics through the lens of cognitive linguistics. We begin by offering a number of examples that show how the use of math in physics differs from the use of math as typically found in math classes. We then explore basic concepts in cognitive semantics to show how humans make meaning with language in general. The critical elements are the roles of embodied cognition and interpretation in context. Then, we show how a theoretical framework commonly used in physics education research, resources, is coherent with and extends the ideas of cognitive semantics by connecting embodiment to phenomenological primitives and contextual interpretation to the dynamics of meaning-making with conceptual resources, epistemological resources, and affect. We present these ideas with illustrative case studies of students working on physics problems with math and demonstrate the dynamical nature of student reasoning with math in physics. We conclude with some thoughts about the implications for instruction.

Responda às questões 1 a 3 a seguir, com base no texto 1 dado.

**Questão 1.** O texto relata uma experiência de atividades de matemática desenvolvida no programa PRIMES. Descreva o que é o programa, qual seu público alvo, onde ele é desenvolvido e qual é a premissa que o entrevistado considera factível.

**Questão 2.** Que tipos de projeto não são considerados bons projetos?

**Questão 3.** Quais as características de um bom projeto para este programa?

Responda às questões 4 e 5 a seguir, com base no texto 2 dado.

**Questão 4.** Descreva em suas palavras a experiência didática descrita do curso de virologia realizado. Mencione nesse texto as características descritas no texto sobre o processo, do método, dos resultados e das formas de avaliação.

**Questão 5.** O texto menciona outros métodos que vêm sendo utilizados, e seus resultados. Sumarize o que o texto descreve e a recomendação principal feita nele a respeito do processo de ensino-aprendizagem em ciências.

Responda à questão 6 a seguir, com base no texto 3 dado.

**Questão 6.** Elabore uma versão em português do texto.